International Journal of Heat and Mass Transfer 1997 vol.40 N5, pages 1191-1196

Explicit, rigorous solutions to two-dimensional heat transfer: Two-component media and optimization of cooling fins

Kacimov A., Obnosov Y.
Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

New analytical solutions to the problem of steady heat conduction from the wall with longitudinal fins to the environment are derived. Within the two media two temperature fields are harmonic functions with rigorous conjugation of temperature and normal flux along the interface between the two components. First, for high values of the ratio e = k1/k2, with k1 and k2 being thermal conductivities of the grooved wall and environment, respectively, we derive the optimal fin contour providing extreme heat flux (total heat dissipation) from the fin surface at prescribed fin cross-sectional area. This optimizer is found in the class of arbitrary curves and both necessary and sufficient extremum conditions are satisfied. The extreme line coincides with the contour of constant hydraulic gradient calculated by Polubarinova-Kochina for a seepage flow under a concrete dam. At arbitrary e the same isoperimetric problem is solved in the class of elliptic fins assuming fin spacing large enough to consider an isolated profile. Two non-trivial local extrema exist depending on e. For arbitrary e the case of long rectangular fins with arbitrary direction of the outer field is studied. Streamline refraction illustrates non-trivial fluxes near the finger tips and roots.